

SPM BULLETIN

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1. EDITOR'S NOTE

This is the first issue after the successful SPM07 meeting. Lively personal impressions of two major participants appear in Section 2: A detailed account by Zdomskyy, including a brief description of the talks, and a concise one by Di Maio. I hope that this will be some sort of compensation for readers who unfortunately could not make it to the meeting, and give us a good reason to look forward to the next meetings.

The Workshop's official web-page is

<http://www.pmf.ni.ac.yu/spm2007/index.html>

The presentations are available at

<http://www.cs.biu.ac.il/~tsaban/SPMC07/slides.html>

Some pictures are available at

<http://www.thesamet.com/spm/photos/main.php>

(more pictures will be added there later).

Boaz Tsaban, boaz.tsaban@weizmann.ac.il

<http://www.cs.biu.ac.il/~tsaban>

2. PERSONAL IMPRESSIONS FROM THE SPM07 MEETING

The atmosphere of the workshop was friendly and stimulating. The rather warm April for central Europe, and the beautiful nature surrounding us added to the positive atmosphere. A welcome party and a banquet were organized in a typical Slavic manner. The participants could enjoy visiting centers of Serbian culture. This was the background of the **III Workshop on Coverings, Selections and Games in Topology**.

The conference was opened by the talk of *V. Fedorchuk*, who presented some results from dimension theory. In particular, $wid = w\text{-}\infty\text{-}C$, and hence all the properties $w\text{-}m\text{-}C$ coincide, where $m \in \mathbb{N}$. Earth is really compact: Fedorchuk is my mathematical great grandfather.

By its spirit, Fedorchuk's talk was close to that of *L. Babinkostova*. These talks demonstrated that selection principles is not only the "zero-dimensional" part of topology, but also has natural connections with dimension theory, and there are promising directions here.

M. Scheepers showed in his talk how many faces the Sakai property $S_1(\Omega, \Omega)$ has. The talk was very impressive: one of the participants told me that he would be happy to have this talk at least one hour longer.

The next speaker, *G. Di Maio* presented a very intensive talk. It was shown how natural it is to bring the notion of abstract boundedness into selection principles. He also showed connections with Arkhangel'ski-Kočinac α_i -properties, and Ramsey theory. And he a musician as well!

M. Mršević presented some current results concerning selection properties of topological (hyper)spaces defined by means of Čech closure operators. Except for the mathematical content of her talk, the participants could enjoy (and join) her greetings to Ljubiša Kočinac on the occasion of his 60th (he looks younger) anniversary.

Some heavy and very interesting stuff about universal elements in some classes of mappings and classes of G -spaces was presented by *S. Iliadis*. The number of definitions and theorems per minute was so big, that one should be in good shape to follow them all. Nonetheless, the main results were presented in a clear way.

P. Blagojević, *T.-L. Costache*, and *M. Joita*'s talks returned me to the reality that the mathematics is very rich and diverse. The main ideas were presented in a very gentle way, which made it very pleasant to listen to these talks. Blagojević started with a nice riddle, and proceeded to obtain a powerful Borsuk-Ulam type theorem. Costache and Joita presented a number of results about C^* -algebras.

L. Kočinac has also demonstrated the quick growth of field of selection principles: Prior to his talk, I heard the terms “regularly varying, slowly varying” sequences only from people doing complex analysis. Some of the presented results are nice: one can have a feeling that it is possible to prove them “on fingers”, but gives up after delicate places are encountered.

In his talk, *D. Georgiou* mainly considered the different topologies on the spaces of continuous functions and compared there properties. Besides the huge variety of results, he did an important job: you can find a quite long list of many classical and important works determining directions in topology in his presentation.

B. Tsaban and *N. Samet* presented a nice one-hour talk consisting of two parts devoted to Ramsey theory. The first part (by Tsaban) contained motivation, all needed definitions and basic results in Ramsey theory. One could enjoy nice presentation accompanied by clear explanations, which resulted in a smooth and accessible introduction to the topic.

The heavy staff was presented by Samet. He made a (successful, to my opinion) effort to bring the participants to the front line of Ramsey Theory and presented some delicate proofs.

The most impressive effort to force the participants to understand a tricky and difficult proof was made by *M. Machura*. He spoke about his result (joint with S. Shelah and B. Tsaban) concerning finite powers of σ -bounded groups. The importance of the results justifies the difficulty of the proof. Macura’s didactic presentation was a work of art.

M. Sakai mainly spoke about local properties of spaces of continuous functions. Some of the results were quite surprising: It was brave of him to dare (and succeed) proving that $C_p(X)$ has the property (\sharp) for every X .

Some results concerning selection principles in relator spaces were presented by *D. Kocev*. He demonstrated that many well-known facts in the realm of topological spaces can be extended to relator spaces. Therefore, these results do not even use that the underlying object is a topological space in a full strength, let alone some separation axioms, etc.

V. Vuksanović gave an overview of the present status of so-called canonical Ramsey theory. In particular, he spoke about generalizations of such notions like the Ellentuck topology, and mentioned some prominent results of Todorcevic and others whose proofs use these generalizations.

Various topologies on the set $C(X)$ of all continuous real-valued functions were considered by *V. Pavlović*. Besides the compact-open one, he considered topologies obtained when a function is identified with its graph, which is treated as a point of some hyperspace. Analogies with the classical results were presented. Typical to his good sense of humor, the end of the talk was really funny.

The following striking characterization of I -favorable spaces was presented by *S. Plewik*: A compact space X is I -favorable if, and only if, X can be represented as

a limit of σ -complete inverse system of compact metrizable spaces with skeletal bonding maps. By its spirit, this result is similar to Shchepins theory of openly generated spaces.

The talk of *P. Kalembe* was devoted to doughnuts, which are natural generalizations of basic open sets in the Ellentuck topology. In particular, some modifications of the *base matrix tree lemma* were presented, and they were used to establish a number of inequalities between cardinal characteristics of doughnuts.

L. Bukovský spoke about properties of spaces of continuous functions defined with help of the quasi-normal convergence: wQN -, QN -spaces, and their modifications. One could enjoy the clear presentation of his results by diagrams.

I talked about joint results with B. Tsaban, characterizing several classes of hereditarily Hurewicz spaces. In the workshop we learned that the one of the main problems from Bukovský's talk is solved by our characterizations. This is yet another demonstration of the importance of such meetings.

The future of the *SPM*-Bulletin was discussed during the SPM Forum. One of the most interesting (to my opinion) suggestions was made by S. Plewik, who proposed to establish a collaboration with the AMS in order to improve the MathSciNet-reviews of the SPM-papers.

Finally, participants had a nice opportunity to learn many interesting facts about the life and work of *D. Kurepa*, in a Round Table dedicated to his centennial. Besides the information available online, I was impressed by the personal recollections of *L. Bukovský* concerning his meetings with *Kurepa*.

Lyubomyr Zdomskyy

The SPM 07 workshop was a great meeting from several points of view:

- (1) Carefully organized (and I believe that this is not easy, especially in Serbia);
- (2) The atmosphere was friendly;
- (3) The level of talks was high;
- (4) I feel at home in the SPM group.

Giuseppe Di Maio

3. RESEARCH ANNOUNCEMENTS

3.1. Coloring ordinals by reals. We study combinatorial principles we call Homogeneity Principle $HP(\kappa)$ and Injectivity Principle $IP(\kappa, \lambda)$ for regular $\kappa > \aleph_1$ and $\lambda \leq \kappa$ which are formulated in terms of coloring the ordinals $< \kappa$ by reals.

<http://arxiv.org/abs/0704.1884>

Jörg Brendle and Sakaé Fuchino

3.2. Long Borel Hierarchies. We show that it is relatively consistent with ZF that the Borel hierarchy on the reals has length ω_2 . This implies that ω_1 has countable cofinality, so the axiom of choice fails very badly in our model. A similar argument

produces models of ZF in which the Borel hierarchy has length any given limit ordinal less than ω_2 , e.g., ω or $\omega_1 + \omega_1$.

<http://www.math.wisc.edu/~miller/res/index.html>

Arnold W. Miller

3.3. Rothberger's property in finite powers. We show that several classical Ramseyan statements, and a forcing statement, are each equivalent to having Rothberger's property in all finite powers.

<http://arxiv.org/abs/0705.0504>

Marion Scheepers

3.4. Special subsets of the reals and tree forcing notions. We study relationships between classes of special subsets of the reals (e.g., meager-additive sets, γ -sets, C''' -sets, λ -sets) and the ideals related to the forcing notions of Laver, Mathias, Miller and Silver.

<http://www.ams.org/journal-getitem?pii=S0002-9939-07-08808-9>

Marcin Kysiak, Andrzej Nowik, and Tomasz Weiss

3.5. All automorphisms of the Calkin algebra are inner. We prove that it is relatively consistent with the usual axioms of mathematics that all automorphisms of the Calkin algebra are inner. Together with a 2006 Phillips–Weaver construction of an outer automorphism using the Continuum Hypothesis, this gives a complete solution to a 1977 problem of Brown–Douglas–Fillmore. We also give a simpler and self-contained proof of the Phillips–Weaver result.

<http://arxiv.org/abs/0705.3085>

Ilijas Farah

3.6. Continuous selections and σ -spaces. Assume that X is a metrizable separable space, and each clopen-valued lower semicontinuous multivalued map Φ from X to Q has a continuous selection. Our main result is that in this case, X is a σ -space. We also derive a partial converse implication, and present a reformulation of the Scheepers Conjecture in the language of continuous selections.

<http://arxiv.org/abs/0705.2867>

Dusan Repovs, Boaz Tsaban, and Lyubomyr Zdomskyy

3.7. On the closure of the diagonal of a T_1 -space. Let X be a topological space. The closure of $\Delta = \{(x, x) : x \in X\}$ in $X \times X$ is a symmetric relation on X . We characterize those equivalence relations on an infinite set that arise as the closure of the diagonal with respect to a T_1 -topology.

<http://arxiv.org/abs/0705.3877>

Maria-Luisa Colasante and Dominic van der Zypen

3.8. Splitting families and the Noetherian type of $\beta\mathbb{N} \setminus \mathbb{N}$. Extending some results of Malykhin, we prove several independence results about base properties of $\beta\mathbb{N} \setminus \mathbb{N}$ and its powers, especially the Noetherian type $Nt(\beta\mathbb{N} \setminus \mathbb{N})$, the least κ for which $\beta\mathbb{N} \setminus \mathbb{N}$ has a base that is κ -like with respect to containment. For example, $Nt(\beta\mathbb{N} \setminus \mathbb{N})$ is never less than the splitting number, but can consistently be that \aleph_1 , 2^{\aleph_0} , $(2^{\aleph_0})^+$, or strictly between \aleph_1 and 2^{\aleph_0} . $Nt(\beta\mathbb{N} \setminus \mathbb{N})$ is also consistently less than the additivity of the meager ideal. $Nt(\beta\mathbb{N} \setminus \mathbb{N})$ is closely related to the existence of special kinds of splitting families.

<http://arxiv.org/abs/0705.4297>

David Milovich

3.9. Even more simple cardinal invariants. Using GCH, we force the following: There are continuum many simple cardinal characteristics with pairwise different values.

<http://arxiv.org/abs/0706.0319>

Jakob Kellner

3.10. A classification of CO spaces which are continuous images of compact ordered spaces. A compact Hausdorff space X is called a CO space, if every closed subset of X is homeomorphic to an open subset of X . Every successor ordinal with its order topology is a CO space. We find an explicit characterization of the class K of CO spaces which are a continuous image of a Dedekind complete totally ordered set. (The topology of a totally ordered set is taken to be its order topology). We show that every member of K can be described as a finite disjoint sum of very simple spaces. Every summand has either form: (1) $\mu + 1 + \nu^*$, where μ and ν are cardinals, and ν^* is the reverse order of ν ; or (2) the summand is the 1-point-compactification of a discrete space with cardinality \aleph_1 .

<http://arxiv.org/abs/0706.1686>

Robert Bonnet and Matatyahu Rubin

4. PROBLEM OF THE ISSUE

The general definitions of the properties considered here are available in [3], but in our specific case there are combinatorial reformulations [1], which we reproduce here for the reader's convenience. For a $g \in \mathbb{Z}^{\mathbb{N}}$, $f \in \mathbb{N}^{\mathbb{N}}$, and $n \in \mathbb{N}$, $|g| \upharpoonright_{[0,n]} \leq f(n)$ means: For each $k \in [0, n)$ (that is, each $k < n$), $|g(k)| \leq f(n)$.

A subgroup G of $\mathbb{Z}^{\mathbb{N}}$ is *Menger-bounded* if, and only if, there is $f \in \mathbb{N}^{\mathbb{N}}$ such that

$$(\forall g \in G)(\exists^\infty n) \ |g| \upharpoonright_{[0,n]} \leq f(n).$$

G^2 is Menger-bounded if, and only if, there is $f \in \mathbb{N}^{\mathbb{N}}$ such that

$$(\forall g_0, g_1 \in G)(\exists^\infty n)(\forall i = 0, 1) \ |g_i| \upharpoonright_{[0,n]} \leq f(n).$$

In [2] we used a weak but unprovable hypothesis to prove that there is a group $G \leq \mathbb{Z}^{\mathbb{N}}$ such that G is Menger-bounded, but G^2 is not Menger-bounded.

A subgroup G of $\mathbb{Z}^{\mathbb{N}}$ is *Rothberger-bounded* if, and only if, for each increasing $h \in \mathbb{N}^{\mathbb{N}}$, there is $\varphi : \mathbb{N} \rightarrow \mathbb{Z}^{<\aleph_0}$ such that:

$$(\forall g \in G)(\exists n) \ g \restriction_{[0, h(n))} = \varphi(n).$$

$g \restriction_{[0, h(n))} = \varphi(n)$ means: For each $k \in [0, h(n))$ (that is, each $k < h(n)$), $g(k) = \varphi(n)(k)$.

Clearly, each Rothberger-bounded group is Menger-bounded.

Problem 4.1. *Does CH imply the existence of a group $G \leq \mathbb{Z}^{\mathbb{N}}$ such that G is Rothberger-bounded but G^2 is not Menger-bounded?*

This problem is raised in [1], and was independently suggested by Ljubša Kočinac during the SPM07 Workshop.

REFERENCES

- [1] M. Machura and B. Tsaban, *The combinatorics of the Baer-Specker group*, Israel Journal of Mathematics, to appear (23 pages).
- [2] M. Machura, S. Shelah, and B. Tsaban, *Squares of Menger-bounded groups*, <http://arxiv.org/abs/mathGN0611353>, submitted.
- [3] L. Babinkostova, Lj. D.R. Kočinac, and M. Scheepers, *Combinatorics of open covers (XI): Menger- and Rothberger-bounded groups*, Topology and its Applications **154** (2007), 1269–1280.

5. UNSOLVED PROBLEMS FROM EARLIER ISSUES

Issue 1. *Is $\left(\frac{\Omega}{\Gamma}\right) = \left(\frac{\Omega}{\Gamma}\right)$?*

Issue 2. *Is $U_{fin}(\mathcal{O}, \Omega) = S_{fin}(\Gamma, \Omega)$? And if not, does $U_{fin}(\mathcal{O}, \Gamma)$ imply $S_{fin}(\Gamma, \Omega)$?*

Issue 4. *Does $S_1(\Omega, \Gamma)$ imply $U_{fin}(\Gamma, \Gamma)$?*

Issue 5. *Is $\mathfrak{p} = \mathfrak{p}^*$? (See the definition of \mathfrak{p}^* in that issue.)*

Issue 6. *Does there exist (in ZFC) an uncountable set satisfying $S_1(\mathcal{B}_\Gamma, \mathcal{B})$?*

Issue 8. *Does $X \notin \text{NON}(\mathcal{M})$ and $Y \notin \text{D}$ imply that $X \cup Y \notin \text{COF}(\mathcal{M})$?*

Issue 9 (CH). *Is $\text{Split}(\Lambda, \Lambda)$ preserved under finite unions?*

Issue 10. *Is $\text{cov}(\mathcal{M}) = \mathfrak{od}$? (See the definition of \mathfrak{od} in that issue.)*

Issue 11. *Does $S_1(\Gamma, \Gamma)$ always contain an element of cardinality \mathfrak{b} ?*

Issue 12. *Could there be a Baire metric space M of weight \aleph_1 and a partition \mathcal{U} of M into \aleph_1 meager sets where for each $\mathcal{U}' \subset \mathcal{U}$, $\bigcup \mathcal{U}'$ has the Baire property in M ?*

Issue 14. *Does there exist (in ZFC) a set of reals X of cardinality \mathfrak{d} such that all finite powers of X have Menger's property $S_{fin}(\mathcal{O}, \mathcal{O})$?*

Issue 15. *Can a Borel non- σ -compact group be generated by a Hurewicz subspace?*

Issue 16 (MA). *Is there an uncountable $X \subseteq \mathbb{R}$ satisfying $S_1(\mathcal{B}_\Omega, \mathcal{B}_\Gamma)$?*

Issue 17 (CH). *Is there a totally imperfect X satisfying $U_{fin}(\mathcal{O}, \Gamma)$ that can be mapped continuously onto $\{0, 1\}^\mathbb{N}$?*

Issue 18 (CH). *Is there a Hurewicz X such that X^2 is Menger but not Hurewicz?*

Issue 19. *Does the Pytkeev property of $C_p(X)$ imply the Menger property of X ?*

Issue 20. *Does every hereditarily Hurewicz space satisfy $S_1(\mathcal{B}_\Gamma, \mathcal{B}_\Gamma)$?*

Issue 21 (CH). *Is there a Rothberger-bounded $G \leq \mathbb{Z}^\mathbb{N}$ such that G^2 is not Menger-bounded?*

Previous issues. The previous issues of this bulletin, which contain general information (first issue), basic definitions, research announcements, and open problems (all issues) are available online, at <http://front.math.ucdavis.edu/search?&t=%22SPM+Bulletin%22>

Contributions. Please submit your contributions (announcements, discussions, and open problems) by e-mailing us. It is preferred to write them in \LaTeX . The authors are urged to use as standard notation as possible, or otherwise give the definitions or a reference to where the notation is explained. Contributions to this bulletin would not require any transfer of copyright, and material presented here can be published elsewhere.

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